

# The Soft-Virtual Higgs Cross-section at N<sup>3</sup>LO and the Convergence of the Threshold Expansion

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We discuss the validity of the soft-virtual approximation and the threshold expansion for the Higgs boson production cross-section at hadron colliders in perturbative QCD up to next-to-next-to-next-to-leading order (N<sup>3</sup>LO).

## 1 Introduction

Studying the properties of the Higgs boson, which was recently discovered by the ATLAS and CMS collaboration at the LHC<sup>2</sup>, demands high precision prediction for experimental results. Furthermore, to be able to distinguish Standard Model (SM) physics from possible deviations a precise theoretical knowledge of the predictions for the experimental outcome is vital. Soon the determination of the strengths of the Higgs bosons interactions will be limited by insufficiently precise predictions.

The Higgs production cross-section at the LHC takes the form

$$\sigma = \sum_{ij} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}(m_H^2, x_1 x_2 s), \quad (1)$$

where  $\hat{\sigma}_{ij}$  are the partonic cross-sections for producing a Higgs boson from partons  $i$  and  $j$ ,  $f_i(x_1)$  and  $f_j(x_2)$  are the corresponding parton distribution functions, and  $m_H$  and  $s$  denote the mass of the Higgs boson and the hadronic centre-of-mass energy, respectively. The largest contribution to the partonic cross-section is given by the gluon-fusion production mode creating a Higgs boson via a top quark loop that is formed via the interaction of two initial state gluons. The relatively light mass of the Higgs boson allows for the calculation of this process in perturbative QCD in the infinite top-quark mass limit. Currently, the gluon-fusion cross-section is known in this fixed order QCD approximation up to next-to-next-to-leading order (NNLO)<sup>6,5</sup> and many additional corrections are available (see ref.<sup>3</sup> for a comprehensive summary).

The largest perturbative uncertainty of the partonic cross-section originates from the missing next-to-next-to-next-to-leading order (N<sup>3</sup>LO) QCD corrections to the gluon fusion production channel. Recently the first term of a threshold expansion, of the N<sup>3</sup>LO gluon fusion channel, was made public in the letter<sup>1</sup>. The result presents a first milestone towards the missing piece and contains the combination of new and previously existing results<sup>4</sup> to the soft-virtual (SV) approximation.

To arrive at a reliable phenomenological prediction it is highly important to understand the limitation of the threshold expansion and draw conclusions about the necessity for further

improvement via the calculation of sub-leading terms or even the full, unexpanded cross-section. In this proceedings we study this uncertainty in the case of the gluon fusion Higgs production cross-section at N<sup>3</sup>LO. We consider lower orders in perturbative QCD to study the convergence behaviour of the expansion for the Higgs cross-section and inspect the impact of the ambiguity due to the truncation of the threshold expansion. Furthermore, we demonstrate that the ambiguity for the SV approximation at N<sup>3</sup>LO is large.

## 2 Threshold Expansion for the Higgs boson cross-section

The probability distribution of a gluon occurring in a proton is steeply falling with its energy and suggests the possibility of performing a fast converging threshold expansion of the gluon fusion Higgs cross-section. Already at NNLO a threshold expansion was performed<sup>7</sup> and was shown to be rapidly converging towards the full result<sup>6</sup>.

Here we study the strong coupling expansion of the heavy top effective theory. In this note we are interested in the effect complementing existing lower order calculations with a threshold expansion at N<sup>n</sup>LO. The threshold approximations and expansions which we will discuss will always contain the full (non-expanded) dependence on terms which enter the result at lower orders in the strong coupling expansion. We will also include the full N<sup>n</sup>LO dependence on renormalisation and factorisation scales as well as the full dependence on those N<sup>n</sup>LO corrections which are generated from higher order corrections to the Wilson coefficient.

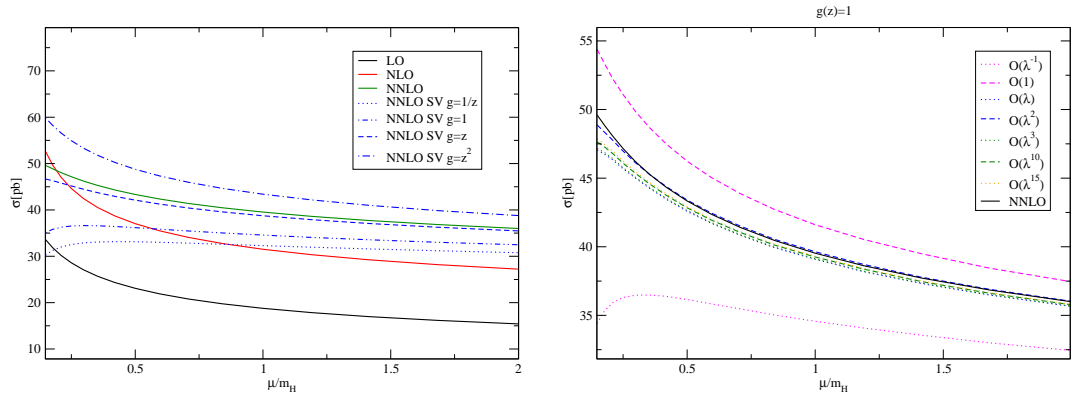
Parametrising the expansion with the variable  $z = \frac{m_H^2}{x_1 x_2 s}$  leads to a series of the partonic cross-section in  $(1 - z)$ .

$$[\hat{\sigma}_{ij}(s, z)]_{threshold} = \sigma_{ij}^{SV} + (1 - z)^0 \sigma_{ij}^{(0)} + (1 - z) \sigma_{ij}^{(1)} + \dots \quad (2)$$

If a series expansion is truncated at a given finite order an unavoidable ambiguity is introduced due to missing higher order terms. To study the impact of truncating the threshold expansion of the Higgs boson cross-section we spuriously insert a function  $g(z)$  satisfying  $\lim_{z \rightarrow 1} g(z) \rightarrow 1$  into eq. 1 such that

$$\sigma = \sum_{i,j} \int dx_1 dx_2 [f_i(x_1) f_j(x_2) z g(z)] \left[ \frac{\hat{\sigma}_{ij}(s, z)}{z g(z)} \right]_{threshold} \quad (3)$$

For all choices of  $g(z)$  the expansion truncated at  $\mathcal{O}((1 - z)^n)$  thus leads to formally equivalent results up to  $\mathcal{O}((1 - z)^{n+1})$ .



(a) Full result up to NNLO and NNLO SV term with different choices for  $g(z)$  as function of  $\mu$  (b) Expansion at NNLO truncated at different  $\lambda = 1 - z$  for  $g(z)=1$

Figure 1 – Threshold approximation for the Higgs boson cross-section at 13 TeV at the LHC

In Fig.1(a) we show the Higgs boson cross-section up to NNLO and the NNLO term including only the SV term as a function of the renormalisation and factorisation scale  $\mu = \mu_R = \mu_F$ . The different lines for the NNLO SV contribution correspond to different choices  $g(z) = \{1, z, z^2, \frac{1}{z}\}$ , respectively.

We note that the variation among the different choices is sizeable and suggests a large impact of sub-leading terms at NNLO. Of the selected choices  $g(z) = z$  represents the closest approximation to the full result at NNLO. Analysing the first sub-leading term of the threshold expansion we find that this choice correctly reproduces the coefficient of the  $\log^n(1-z)$ , where  $n$  is the largest appearing power. We found similar behaviour when analysing the NLO term.

In Fig.1(b) we show the NNLO threshold approximation up to various orders in the expansion for the choice  $g(z) = 1$  as a function of  $\mu$ . One can clearly see that the first and second term suffer from a large discrepancy compared to the full result. Furthermore, we observe that the quality of the expansion is rather independent of the chosen scale  $\mu$ . We found similar behaviour for other choices of  $g(z)$ .

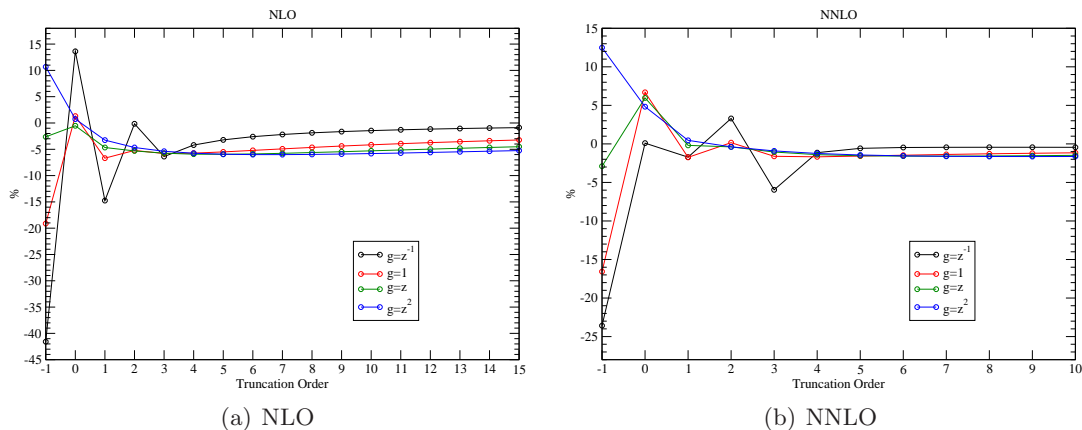


Figure 2 – Percent difference of the threshold expansion to the full Higgs boson cross-section at NLO and NNLO at 13 TeV at the LHC evaluated at  $\mu = \frac{m_H}{2}$  as a function of the order where the series is truncated. Different lines correspond to different choices for the function  $g(z)$ .

In Fig.2 we present the Higgs boson cross-section at NLO and NNLO evaluated at  $\mu = \frac{m_H}{2}$  for the same choices of  $g(z)$  as above as a function of the order where the threshold expansion is truncated. We observe sizeable changes of the prediction comparing the first and second term for all our choices of  $g$ . However the convergence pattern observed for different choices of  $g$  are rather different. Indeed while for lower orders in the expansion,  $g = 1/z$  is the worst choice, it becomes the best when up to 5 or more terms are included. This is particularly true at NLO, where the other choices only reproduce the full result within about 5%. The same effect, though much reduced in size, is also observed at NNLO and is directly related to the fact that the choices  $g = z^n$ , for  $n \geq 0$ , introduce a further damping of the gluon luminosity away from threshold, which is compensated by introducing a factor  $1/z^{n+1}$  into the partonic cross section. This factor which increases the sensitivity to the high energy regime is subsequently expanded around the threshold and its effect is therefore lost completely in the SV and still to some extent when further terms in the threshold expansion are taken into account. At NNLO it appears that the effect of this factor is much smaller than at NLO and we may expect this to hold also at N<sup>3</sup>LO.

In Fig.3 we present the SV Higgs cross-section at N<sup>3</sup>LO for the same choices of  $g(z)$  as a function of  $\mu$ . Again the various choices lead to drastically different predictions for the Higgs boson cross-section. We observe that the hierarchy of the lines is changed compared to lower orders. The choices  $g(z) = \{\frac{1}{z}, 1, z\}$  at  $\frac{m_H}{2}$  are in agreement with the scale uncertainty at NNLO. The large spread of the different choices suggests the possibility for large corrections due

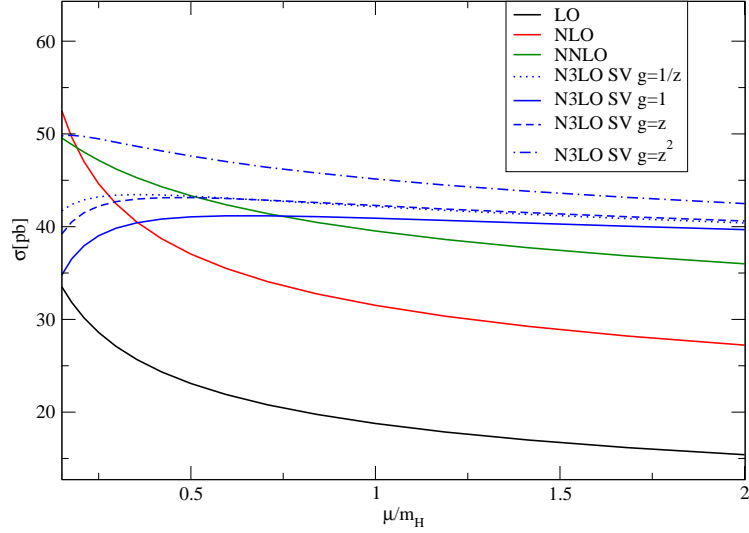


Figure 3 – The gluon-fusion cross-section at 13 TeV at the LHC as a function of  $\mu = \mu_R = \mu_F$  up to LO (black), NLO (red), NNLO (green) and soft-virtual N<sup>3</sup>LO (blue). The N<sup>3</sup>LO SV approximation is modified with different functions  $g(z)$ .

to sub-leading terms at N<sup>3</sup>LO. Given the experience at lower orders we expect that only a few sub-leading terms in the threshold expansion are required to obtain a significant improvement to an approximation of the N<sup>3</sup>LO cross-section and consequently to the predictions for LHC observables.

### 3 Conclusion

The rapidly increasing experimental precision of Higgs cross-section measurements raises an urgent demand for the improvement of the theoretical prediction for the inclusive Higgs boson cross-section at the LHC. With the recent publication of the first term in the threshold expansion of the N<sup>3</sup>LO gluon-fusion QCD cross-section an important step in this direction was taken. In this proceedings we have analysed the quality of the threshold expansion. We find that the expansion is converging fast at lower orders in QCD perturbation theory and expect to find similar behaviour at N<sup>3</sup>LO. We studied the uncertainty introduced due to the truncation of the threshold expansion at NLO, NNLO and N<sup>3</sup>LO and conclude that at least several terms in the expansion are necessary in order to infer reliable predictions for LHC measurements and improve upon the current status. We conclude that the calculation of further terms in the threshold expansion and even the full Higgs boson cross-section at N<sup>3</sup>LO is highly desirable.

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